TEST OF AN AN ARTIFICIAL AERIAL TELEPHONE LINE AT A FREQUENCY OF 750 CYCLES PER SECOND

BY

A. E. KENNELLY and F. W. LIEBERKNECHT

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ABSTRACT OF PAPER

An artificial aerial telephone line 800 km. (500 miles) and 2 wavelengths long, is described, and a series of tests upon it is reported, under three terminal conditions at the receiving end, and with about 1.7 volts of e.m.f., alternating at 760 ~, impressed at the sending end. The results obtained are compared with those deducible theoretically, and they are shown to support the regular hyperbolic-function theory.

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It is the purpose of this paper to describe in detail a series of measurements of voltage and current made, in February, 1913, over an artificial telephone line 800 km. (nearly 500 miles) long. Although it is possible to compute the behavior of such a line when its electrical constants are known, yet this is the first time, so far as the authors are aware, that an artificial aerial line has been measured both as to amplitude and phase of voltage and current, with less than 2 volts of 750 cycles-per-second frequency impressed at the sending end.

The artificial line used in these measurements is the one permanently installed in the Graduate School of Applied Science at Harvard University, and described in our paper before the Boston Convention of the A. I. E. E. last year, in relation to a-c. power-transmission.[1] In order to convert this artificial line, originally designed as a power-transmission line, into an aerial telephone line, small non-inductive resistances were uniformly inserted in the line, one to each section.[2] These resistances added 100 ohms per section of 80 km., or increased the nominal linear resistance from about 0.3 ohm per wire-km.,

1. *Measurements of Voltage and Current over a Long Artificial Power-Transmission Line at 25 and 60 Cycles per Second*, by A. E. Kennelly and F. W. Lieberknecht. PROC. A. I. E. E., June, 1912, p. 805.

2. These resistances were made of about 6 meters of No. 32 A. W. G. *Ia-Ia* (0.2 mm. 0.008 inch bare diam.) double-cotton covered resistance wire, and were wound non-inductively on fiber strips 8 cm. long, 2.5 cm. wide, and 0.3 cm. thick, soldered to terminals on the same, and covered in solid paraffin wax. These resistances carry 0.5 ampere without serious heating.

(0.48 ohm per wire-mile), to about 1.55 ohms per wire-km. (2.5 ohms per wire-mile), corresponding approximately to one of a pair of copper overhead telephone wires of copper 3.75 mm. (148 mils) in diameter. That is, one and the same artificial line of coils and condensers can be made to serve either as an artificial aerial power-transmission line, or as an artificial aerial telephone line, by respectively cutting out or in the non-inductive resistance attached to each section, thereby changing the line for low frequencies from about No. 0 B. & S. gage (8.3 mm. 0.325 in. diameter) to No. 7 B. & S. gage (3.65 mm. 0.144 in. diameter).[3] The total available length of line in each case is 2400 km. (1491 miles), only 800 km. (497 miles) being employed in the tests here reported.

For the constructive details of the line reference may be made to our paper of last year already cited.[4] It may suffice to say at

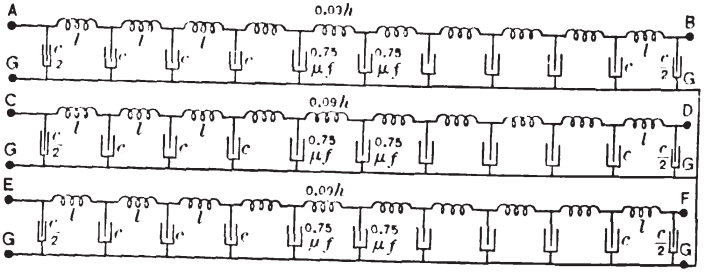

FIG. 1.—DIAGRAM OF CONNECTIONS OF ARTIFICIAL LINE

this time that the line used was a Π-line of ten sections, with ground return, the connections being indicated in Fig. 1. The constants for the line appear in Tables I and II. In the design and construction of an artificial aerial telephone line *per se*, not intended to serve also as a power-transmission artificial line, it would be unnecessary to use as much copper as in the artificial line here described. It would also be desirable to subdivide the inductance and capacity into smaller but more numerous sections for frequencies much above 1000 cycles per second. In the line

3. The lineactually behaved, at telephonic frequency, like one of a pair of No. 9 A. W. G. copper wires (2.9 mm., 0.114 in., diam.), owing to the combined influences of capacity between the layers of the coil winding, and the lumpiness of the line-section. See Table VI.

4. See also an article "Artificial Power-Transmission Line," by A. E. Kennelly and H. Tabossi, *Electrical World*, New York, Feb. 17, 1912, giving the constructive details of the artificial line.

here described, there were five sections per actual wave-length, at 760 cycles per second.

It will be observed that whereas the apparent inductance of the artificial line is the same at both telephonic and lighting frequencies, the apparent leakance is more than ten times greater at the telephonic frequency while the apparent resistance is nearly four times less, and the apparent capacitance 11.3 per cent less. In other words, the effect of increasing the testing frequency on the artificial line from 60 to 760 cycles per second, is to change all the constants except the inductance. Each coil

TABLE I

ELECTRICAL CONSTANTS OF TEN-SECTION ARTIFICIAL LINE AT 60 ~

	Resistance at 20° cent. ohms	Inductance henrys	Capacitance microfarads	Leakance micromhos	Hyp. angle hyps.
Whole line....	1256	0.913	7.5	120	1.08 /71° .5
Per section..	125.6	0.0913	0.75	1.2	0.108 /71° .5
Per wire-km.	1.57	1.14×10^{-3}	9.38×10^{-3}	0.15	0.00136 /71° .5
Per wire-mile	2.53	1.84×10^{-3}	15.1×10^{-3}	0.24	0.00219 /71° .5

TABLE II

ELECTRICAL CONSTANTS OF ARTIFICIAL LINE AT 760 ~

	Apparent resistance at 20° cent. ohms	Apparent inductance henrys	Apparent capacitance microfarads	Apparent leakance micromhos	Hyperbolic angle Hyps. /	Surge impedance ohms /
Whole line..	330	0.9136	6.656	1660	12.94 /79° .5	465 \11°
Per section.	33	0.0914	0.6656	166	1.294 /79° .5	465 \11°
Per wire-km..	0.413	1.14×10^{-3}	8.32×10^{-3}	2.075		
Per wire-mile...	0.665	1.84×10^{-3}	13.4×10^{-3}	3 34		

behaves as though its resistance were 33 ohms instead of 125.6. Its condensers (paper dielectric) behave as though their capacitance had fallen 11.3 per cent while their leakance had risen some ten times. It is known that paper condensers are subject to such changes with rise of frequency.[5] The great apparent reduction in coil resistance can, however, only be attributed to the effect of capacity between the layers of the windings. This

5. "The Capacity and Phase Difference of Paraffined Paper Condensers as Functions of Temperature and Frequency," by Frederick W. Grover, *Bulletin of the Bureau of Standards*, Vol. VII, No. 4, February 28, 1911.

deduction was supported by tests made on the coils alone, at telephonic frequency.

Generator. The alternator used was a Cahill multifrequency generator employing a number of stator armatures side by side, with internal rotor field magnets on the same shaft, giving respectively 2, 4, 8, 16, 32 and 64 cycles per revolution. It was directly coupled through a leather-strip coupling to a 500-volt, four-kw., four-pole direct-current motor. Each of the component alternator armatures was rated at about 500 watts output. Only two of the armatures were used, namely, those giving 2 and 32 cycles per shaft revolution. When running at 1425 rev. per min. or 23.75 rev. per min., these armatures delivered frequencies at 47.5 ~ and 760~ respectively. The former was used to supply a Frahm vibrator frequency indicator, and the latter for the test of the artificial line. When the frequency meter indication was steady at 47.5 ~, the artificial-line frequency was known to be 760 ~. By leading the shunt-field circuit of the

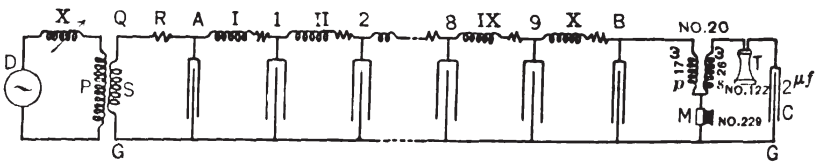

FIG. 2.—CIRCUIT CONNECTIONS OF ARTIFICIAL LINE

driving motor into the artificial-line room, an adjustable field rheostat in this room enabled the driving-motor speed to be kept constant, and thus also the alternator frequencies.

Frequency. The reason for selecting 760 cycles per second for testing the artificial telephone line was that this is near to the standard telephone frequency of 5000 radians per second (795.8 ~) which has been accepted internationally. It is also known to be near to the fundamental frequency of the diaphragms of the standard receiving telephone used.[6]

Circuit Connections. The circuit connections of the artificial line are indicated in Fig. 2. The alternator stator armature *D* delivered 40 volts at 760 ~ in the testing room. A step-down

6. " The Impedance of Telephone Receivers as Affected by the Motion of Their Diaphragms," by A. E. Kennelly and G. W. Pierce, *Proc. Am. Ac. of Arts and Sci.*, Vol. XLVIII, No. 6, September, 1912, p. 113; also *Electrical World*, New York, September 14, 1912; and " The Humming Telephone," by A. E. Kennelly and W. L. Upson, *Proc. Am. Phil. Soc.*, 1908.

transformer, $P\ S$, lowered the voltage to about 1.75 volts at secondary terminals $G\ Q$. The adjustment was made by inserting an adjustable reactance X in the primary circuit, partly for the purpose of reducing upper harmonics in the wave of secondary voltage. The voltage wave-form was somewhat peaked. That is, a symmetrical third harmonic was perceptible; but no analysis of the wave form was attempted. The secondary terminal Q was connected to the end A of the artificial line through a non-inductive resistance R of 100 ohms. The other secondary terminal G was connected to the ground return of the artificial line, and was also carefully grounded.

Three series of tests were made, namely, with the distant end of the line (1) freed, or open, (2) grounded, and (3) connected to ground through a standard subscriber's set.

The third condition is indicated in Fig. 2.

Telephone circuits actually employ metallic return, and not ground return; but all tests on ground-return artificial lines are immediately interpretable on metallic-return or loop basis. These tests correspond to applying 3.5 volts at the sending end of a loop, and two subscriber's sets in series at the receiving end.

Method of Measurement and Apparatus Used

The potential and current along the line in the steady state were measured by means of a Drysdale a-c. potentiometer, using a Duddell vibration galvanometer. The Drysdale instrument was described with some detail in our A. I. E. E. paper of last year, above mentioned. Since the potential on the line nowhere exceeded 1.8 volts, the potentiometer was able to read the potential at any junction point directly, without the intervention of a multiplier or step-down transformer. In order to find the current strength, the drop of potential in the 100-ohm inserted section-resistances was measured.

The Duddell vibration galvanometer[7] was tuned to the testing frequency and operated with the aid of an arc lamp. The spot of light was thrown on a white wall several meters away, so that it was easy to see when balance was secured and the spot of light brought to rest, from almost any part of the testing room.

The 760-cycle-per-second voltage was raised to nearly 140 volts by a step-up transformer, and impressed on the split-

7. "A Bifilar Vibration Galvanometer," by W. Duddell, *Proc. Physical Society*, London, May 14, 1909 and *The Electrician*, London, Vol. LXIII, pp. 620–622, July 30, 1909.

phase angle measurer of the instrument. The phase balance required about 0.043 microfarad and 340 ohms, inserted in the associated shunt circuit of the split-phaser. This split-phase balance had to be checked and adjusted occasionally.

Method of Observation

In making a series of tests along the line, the impressed e.m.f. and frequency at A, Fig. 2, were kept as nearly constant as possible. One observer watched the Frahm frequency meter, another changed the potentiometer leads along the line and watched the vibration-galvanometer arc-lamp. A third adjusted the potentiometer balance for amplitude and phase. Each voltage could ordinarily be measured to 0.001 volt, or closer, in amplitude and to a tenth of a degree in phase. This voltage represented either a potential from some junction point on the line to ground, or the potential drop in a section 100-ohm resistance. In working out the results, they were all reduced by simple proportion to what would have been observed if the impressed e.m.f. at A had been just $1.00\ /0°$ volt instead of the 1.7 volts, or thereabouts, actually measured. This reduction was made for greater simplicity in presenting the results.

Results

The results obtained are indicated in Tables III, IV and V for the condition, at the B end, of grounded, open, and through subscribers' set, respectively. Referring to Table III, the first column denotes the test-points along the line. Arabic numerals designate junction-points between sections, and Roman numerals mid-section points (see Fig. 2). The distances in kilometers of these points from the distant end of the line appear in column 2. The hyperbolic angle δ, of each point from B, is given in the next column, expressed as a complex quantity,[8] the real component being a hyperbolic angle, expressible in hyperbolic radians, and the imaginary, or j-component, a circular angle expressed in circular radians. This j-component measures the wave phase-length of the line at the frequency considered. In this case, the wave-length of the whole line AB is shown to be 12.723 circular radians. It is convenient for many purposes, however, to divide the j-component by $\pi/2$, or 1.5708, in order

8. "The Application of Hyperbolic Functions to Electrical Engineering Problems," by A. E. Kennelly, Universityof London Press, 1912, Chap. V.

to reduce this circular angle to quadrant measure. This reduction appears in the fourth column. The whole line AB subtends a complex hyperbolic angle $2.350 + j\,12.723 = 12.94\,/79.5$ hyps., of which the j-component is 8.1 in quadrant measure. That is, the wave phase-length of the line is 8.1 quadrants, or a little more than 2 complete revolutions. In other words, the artificial line at this frequency covers a little over 2 complete wave-lengths, and the equivalent smooth line corresponding to the artificial line of junctions, would also cover a little over 2 complete wave-lengths. This hypothetical equivalent smooth line which would conform electrically to the lumpy artificial line at junction points, may be briefly denoted as the *imitated line.*

The fifth column of Table III gives the hyperbolic sine of the angle δ at the respective artificial-line junction points.[9] The potential along the artificial line from junction to junction, and on the imitated line at all points, follows sinh δ in simple proportion. The voltage E' so computed, appears in column 6, starting with $1.00\,/0°$ volt at sinh $\delta = 5.2\,/9°.2$, phase angles appearing as degrees and decimal of a degree. Column 7 gives the corresponding observed junction-potential in magnitude and phase with the aid of the potentiometer, taking the initial impressed voltage as $1.00\,/0°$ volt. It will be noticed that the agreement between the computed voltage E' and the observed voltage E is satisfactory, the magnitudes usually agree within 5 per cent and the phase within 5 deg. A discrepancy of 5 deg. may seem considerable in itself, but the total change of voltage phase along the line is about 720 deg.

Column 8 gives the hyperbolic cosines of the mid-section angles It has been shown[10] that the current in any Π-section of an artificial line, when multiplied by the hyp. cosine of a semi-section, agrees with the current on the imitated line at the corresponding point; also that the current at any point of a smooth line is directly proportional to the hyp. cosine of the angle from the distant end. Column 9 gives in this way the computed current at the points on the imitated line corresponding to mid-

9. Tables of complex hyperbolic functions expressed as sinh, cosh, or tanh of $(x + jq)$ from $x = 0$ to $x = 10$, and $q = 0$ to $q = \infty$, by steps in each of 0.05, have been computed, and are in course of publication by the Harvard University Press, together with large-scale interpolation charts.

10. " The Distribution of Voltage and Current over Artificial Lines in the Steady State," by A. E. Kennelly, *Electrical World*, New York, August 10, 1912.

TABLE III

LINE GROUNDED AT B

Point	Distance from B km.	Hyperbolic angle from B: $\delta = x + jy$ hyps. radians	quadrants	sinh δ	Voltage along Line: E' computed volts	Voltage along Line: E observed volts	cosh δ	Current strength along line: I' computed	Current strength along line: I observed milliamperes	Current strength along line: $I \cosh \kappa$ reduced
A	800	$2.350 + j12.723$	$j8.10$	$5.20 / 9°.2$	$1.000/0°$	$1.000/0°$	$5.288/8°.8$	$2.227/12°$	$2.227/12°$	$2.227/12°$
I	760	$2.233 + j12.087$	$j7.695$				$4.710\backslash 27°.1$	$1.984\backslash 23°.9$	$2.454\backslash 32°$	$1.995\backslash 27°$
1	720	$2.115 + j11.451$	$j7.290$	$4.165\backslash 64°.2$	$0.801\backslash 73°.4$	$0.797\backslash 84°$				
II	680	$1.998 + j10.815$	$j6.805$				$3.632\backslash 108°.1$	$1.531\backslash 104°.9$	$1.953\backslash 114°.2$	$1.588\backslash 109°.2$
2	640	$1.880 + j10.179$	$j6.480$	$3.275\backslash 134°.6$	$0.630\backslash 143°.8$	$0.625\backslash 152°.3$				
III	600	$1.763 + j\ 9.543$	$j6.075$				$2.964\backslash 173°.6$	$1.247\backslash 170°.4$	$1.538\backslash 180°$	$1.250\backslash 175°$
3	560	$1.645 + j\ 8.906$	$j5.670$	$2.545\backslash 212°.3$	$0.490\backslash 221°.5$	$0.482\backslash 228.2$				
IV	520	$1.528 + j\ 8.270$	$j5.265$				$2.181\backslash 244°$	$0.919\backslash 240°.8$	$1.150\backslash 255°.7$	$0.935\backslash 250°.7$
4	480	$1.410 + j\ 7.634$	$j4.860$	$2.158\backslash 281°.8$	$0.415\backslash 290°.7$	$0.427\backslash 302°$				
V	440	$1.293 + j\ 6.998$	$j4.455$				$1.847\backslash 323°.5$	$0.778\backslash 320°.3$	$0.923\backslash 332°.2$	$0.750\backslash 327°.2$
5	400	$1.175 + j\ 6.362$	$j4.050$	$1.466/5°.4$	$0.282\backslash 3°.8$	$0.286\backslash 5°$				
VI	360	$1.058 + j\ 5.726$	$j3.645$				$1.525\backslash 25°.9$	$0.642\backslash 22°.7$	$0.780\backslash 34°$	$0.634\backslash 29°$
6	320	$0.940 + j\ 5.089$	$j3.240$	$1.428\backslash 73°.5$	$0.275\backslash 82°.7$	$0.271\backslash 92°.3$				
VII	280	$0.823 + j\ 4.453$	$j2.835$				$0.955\backslash 111°.4$	$0.402\backslash 108°.2$	$0.510\backslash 121°.2$	$0.415\backslash 116°.2$
7	240	$0.705 + \ 3.817$	$j2.430$	$0.994\backslash 126°.5$	$0.491\backslash 135°.7$	$0.199\backslash 143°$				
VIII	200	$0.588 + \ 3.181$	$j2.025$				$1.177\backslash 178°.7$	$0.496\backslash 175°.5$	$0.618\backslash 185°$	$0.502\backslash 180°$
8	160	$0.470 + j\ 2.545$	$j1.620$	$0.749\backslash 237°.9$	$0.144\backslash 247°.1$	$0.140\backslash 253°.7$				
IX	120	$0.353 + j\ 1.909$	$j1.215$				$0.484\backslash 224°.8$	$0.204\backslash 221°.6$	$0.279\backslash 226°$	$0.227\backslash 221°$
9	80	$0.235 + j\ 1.272$	$j0.810$	$0.982\backslash 274°.2$	$0.1889\backslash 283°.4$	$0.1965\backslash 294°$				
X	40	$0.118 + j\ 0.636$	$j0.405$				$0.813\backslash 355°$	$0.342\backslash 351°.8$	$0.439\backslash 0°$	$0.356\backslash 355°$
B	0	$0. \quad + j\ 0$	$j0$	$0.$	$0.$	$0.$	$1.000/0°$	$0.421\backslash 356°.8$	$0.439\backslash 0°$	$0.439\backslash 360°$

section points on the artificial line. Column 10 gives the observed section current I, and the last column gives $I \cosh u$, $\cosh u$ being for this line, the vector $0.813 \angle 5°$. The agreement should be exact, theoretically speaking, between the entries in columns 9 and 11 under I' and $I \cosh u$ respectively. It is not exact in the case considered, but is nevertheless satisfactory.

Table IV gives the corresponding computed and observed potentials and currents with the distant end free. Here the free end of the line constitutes an initial line angle of $j\,\pi/2$ or $j\,1.5708$ radians at B, or just 1 quadrant, which has been added

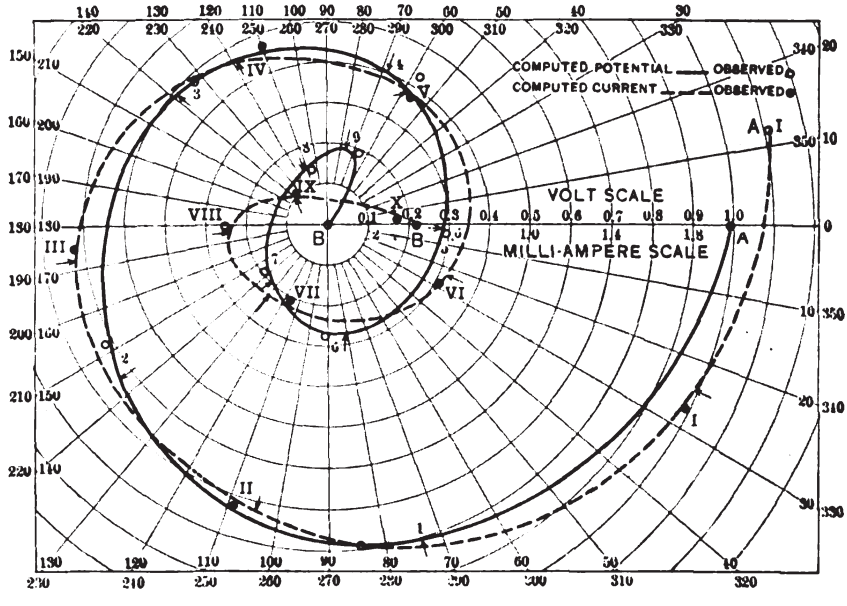

FIG. 3.—LINE GROUNDED AT DISTANT END. VECTOR DIAGRAM OF VOLTAGE AND CURRENT

to all the angles. The voltages follow the sines, and the currents the cosines, of the angles from point to point. The agreement is again satisfactory, although by no means exact, between E' and E, also between I' and $I \cosh u$.

Table V gives a similar comparison of computed and observed potentials and currents for the case of the line grounded at the distant end through a subscriber's set which offered an impedance of $1570 \angle 70°$ ohms, and which therefore subtended a terminal angle of $\tanh^{-1}\left(\frac{1570 \angle 70°}{465 \angle 11°}\right) = 0.043 + j\,1.285$ hyp.; or, with

TABLE IV

LINE FREED AT B

Point	Distance from B km.	Hyperbolic angle from B: $\delta = x + jy$ hyps. radians	quadrants	sinh δ	Voltage along Line: E' computed volts	E observed volts	cosh δ	Current strength along line: I' computed	I observed milliamperes	$I \cosh u$ reduced
A	800	2.350 + j14.294	j9.10	5.288\8°.8	1.000\0°	1.000\0°	5.20\9°.2	2.100\13°	2.100\13°	2.100\13°
I	760	2.233 + j13.658	j8.695				4.717\26°.7	1.905\22°.9	2.286\31°	1.858\26°
1	720	2.115 + j13.022	j8.290	4.109\64°.2	0.777\73°.0	0.7605\78°				
II	680	1.998 + j12.386	j7.805				3.743\107°.2	1.512\103°.4	1.917\109°.9	1.559\104°.9
2	640	1.880 + j11.750	j7.480	3.283\137°.5	0.6209\146°.3	0.6060\151°.8				
III	600	1.763 + j11.114	j7.075				2.753\176°.6	1.112\172°.8	1.385\176°.2	1.126\171°.2
3	560	1.645 + j10.477	j6.670	2.609\208°.9	0.4934\217°.7	0.4857\219°.1				
IV	520	1.528 + j 9.841	j6.265				2.380\248°.5	0.9613\244°.7	1.183\253°.8	0.962\250°.8
4	480	1.410 + j 9.205	j5.860	1.950 285°.7	0.3688 294°.5	0.3671\300°.1				
V	440	1.293 + j 8.569	j5.455				1.806\312°.2	0.7294\308°.4	0.8943\320°.0	0.727\315°
5	400	1.175 + j 7.933	j5.050	1.773\356°.3	0.3353 5°.1	0.3358\3°.2				
VI	360	1.058 + j 7.297	j4.645				1.370\38°.0	0.5533\34°.2	0.6874 \40°.2	0.559\35°.2
6	320	0.940 + j 6.660	j4.240	1.129\63°.0	0.2135\71°.8	0.2175\72°.9				
VII	280	0.823 + j 6.024	j3.835				1.247\113°.2	0.5036\109°.4	0.6942\108°	0.564\103°
7	240	0.705 + j 5.388	j3.430	1.088 153°.4	0.2058\162°.2	0.2052\164°.1				
VIII	200	0.588 + j 4.752	j3.025				0.624\172°.8	0.2520\169°.0	0.3172\177°.1	0.258\172°.1
8	160	0.470 + j 4.116	j2.620	0.9547 196°.0	0.1805 204°.8	0.177 \203°.9				
IX	120	0.353 + j 3.480	j2.215				1.012 \263°.5	0.4088\259°.7	0.528\269°.5	0.429 264°.5
9	80	0.235 + j 2.843	j1.810	0.3854 322°.8	0.0729 \331°.6	0.0726 \331°				
X	40	0.118 + j 2.207	j1.405				0.603 \279°.1	0.2435\275°.3	0.303\282°.2	0.246 \277°.2
B	0	0. + j 1.571	j1.000	1.000\0°	0.1891\8°.8	0.191\6°.7	0.	0.	0.	0.

the j-component in quadrant measure, $0.043 + j0.818$, the surge-impedance of the artificial line being $465 \backslash 11°$ ohms. All the angles of Table III are here increased by these amounts. The potentials are then simply proportional to sinh δ, and the currents to cosh δ, as before. A satisfactory agreement is found between the columns under E' and E; and also under I' and I cosh u.

Figs. 3, 4 and 5 are vector charts, corresponding respectively to Tables III, IV and V. Referring to Fig. 3, with Table III, it will be seen that the potential at A commences at $1.0 \angle 0°$ volt. The continuous curve then traces the hyperbolic-sine locus of

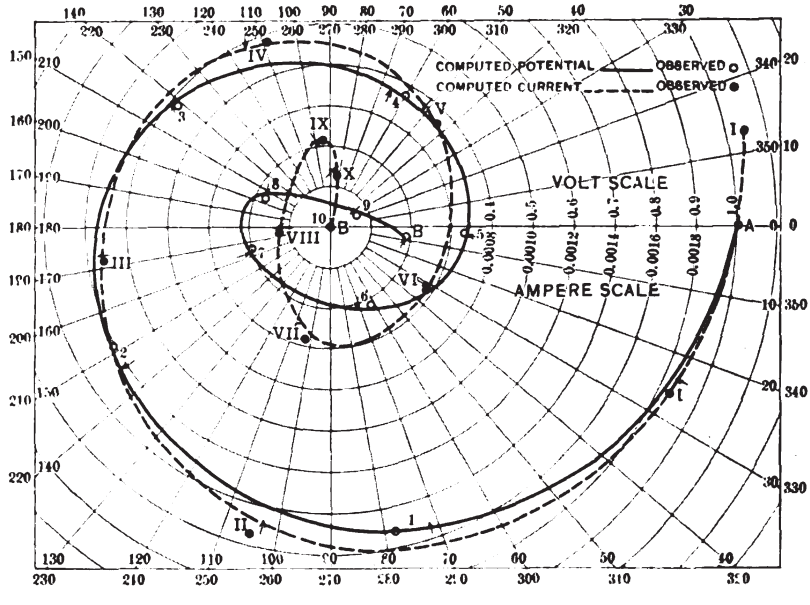

FIG. 4.—LINE FREED AT DISTANT END. VECTOR DIAGRAM OF VOLTAGE AND CURRENT

potential along the imitated line, both in magnitude and phase, it being understood that equal distances in kilometers do not generally correspond to equal phase differences. The arrowheads, intersecting the potential curve, mark the points on the imitated line at 80-km. intervals, corresponding to the junctions of the artificial-line sections. A small circle marks the corresponding vector potential observed. The arrowheads and circles should coincide, theoretically. Similarly, the broken line traces the computed hyperbolic-cosine locus of the vector-current along the imitated line, arrowheads indicating points at 80-km. intervals corresponding to mid-sections on the artificial line.

TABLE V

LINE GROUNDED AT B THROUGH SUBSCRIBER'S SET

Point	Distance from B km.	Hyperbolic angle from B: $\delta = x + jy$ hyps. radians	quadrants	sinh δ	Voltage along Line: E' computed volts	E observed volts	cosh δ	Current strength along line: I' computed	I observed milliamperes	$I \cosh u$ reduced
A	800	$2.393 + j14.008$	$j8.918$	$5.520/82°.2$	$1.000/0°$	$1.000/0°$	$5.477/81°$	$2.099/12°$	$2.099/12°$	$2.099/12°$
I	760	$2.276 + j13.372$	$j8.513$							
1	720	$2.158 + j12.736$	$j8.108$	$4.272/9°.7$	$0.774\backslash 72°.5$	$0.762\backslash 78°.6$	$4.976/44°.5$	$1.907\backslash 24°.5$	$2.362\backslash 31°$	$1.920\backslash 26°$
II	680	$2.041 + j12.100$	$j7.703$				$3.846\backslash 26°.5$			
2	640	$1.923 + j11.464$	$j7.298$	$3.463\backslash 64°.1$	$0.6274\backslash 146°.3$	$0.616\backslash 151°.6$		$1.474\backslash 95°.5$	$1.935\backslash 108°.8$	$1.573\backslash 103°.8$
III	600	$1.806 + j10.828$	$j6.893$				$2.965\backslash 99°.9$			
3	560	$1.688 + j10.191$	$j6.488$	$2.704\backslash 133°.8$	$0.4899\backslash 216°.0$	$0.483\backslash 219°.1$		$1.136\backslash 168°.9$	$1.427\backslash 176°.7$	$1.160\backslash 171°.7$
IV	520	$1.571 + j\ 9.557$	$j6.083$				$2.469\backslash 171°.7$			
4	480	$1.453 + j\ 8.919$	$j5.678$	$2.077\backslash 211°$	$0.382\backslash 293°.4$	$0.379\backslash 301°.8$		$0.9462\backslash 240°.7$	$1.218\backslash 254.6°$	$0.990\backslash 249°.6$
V	440	$1.336 + j\ 8.283$	$j5.273$				$1.837\backslash 243°.5$			
5	400	$1.218 + j\ 7.647$	$j4.868$	$1.826\backslash 280°.6$	$0.3308\backslash 2°.8$	$0.334\backslash 1°$		$0.704\backslash 312°.5$	$0.851\backslash 320°.7$	$0.692\backslash 315°.7$
VI	360	$1.101 + j\ 7.011$	$j4.463$				$1.534\backslash 324°.9$			
6	320	$0.983 + j\ 6.374$	$j4.058$	$1.152\backslash 253°.5$	$0.2087\backslash 75°.7$	$0.2071\backslash 78°.1$		$0.5879\backslash 33°.9$	$0.7386\backslash 40°.1$	$0.601\backslash 35°.1$
VII	280	$0.866 + j\ 5.738$	$j3.653$				$1.299\backslash 27°.9$			
7	240	$0.748 + j\ 5.102$	$j3.248$	$1.236\backslash 75°.1$	$0.2239\backslash 157°.3$	$0.2295\backslash 159°.8$		$0.4978\backslash 96°.9$	$0.6602\backslash 103°.5$	$0.537\backslash 98°.5$
III	200	$0.631 + j\ 4.436$	$j2.843$				$0.718\backslash 113°.8$			
8	160	$0.513 + j\ 3.830$	$j2.438$	$0.834\backslash 119°.4$	$0.1511\backslash 201°.6$	$0.1537\backslash 199°$		$0.2752\backslash 182°.8$	$0.3481\backslash 188°.2$	$0.283\backslash 183°.2$
IX	120	$0.396 + j\ 3.194$	$j2.033$				$1.078\backslash 178°.7$			
9	80	$0.278 + j\ 2.557$	$j1.628$	$0.631\backslash 243°.3$	$0.1143\backslash 325°.5$	$0.1115\backslash 333°$		$0.4131\backslash 247°.7$	$0.5548\backslash 258°$	$0.443\backslash 253°$
X	40	$0.161 + j\ 1.921$	$j1.223$				$0.369\backslash 204°.8$			
								$0.1414\backslash 273°.8$	$0.1868\backslash 274°.3$	$0.152\backslash 269.3°$
B	0	$0.043 + j\ 1.285$	$j0.818$	$0.979\backslash 275°.3$	$0.1773\backslash 357°.5$	$0.1828\backslash 352°$	$0.293/8°$	$0.1123\backslash 61°$	$0.115\backslash 62°$	$0.115\backslash 62°$

The black dot circles mark the reduced current measurements. These arrowheads and black dots should also coincide, according to theory.

It will be seen that the current leads the impressed e.m.f. at *A* by 12 deg. As we advance down the line, both potential and current fall behind the phase of the impressed terminal e.m.f. The change in phase is about 720 degrees for potential, and 732 degrees for current; or about 0.9 deg. per km.

In the case of Fig. 4 and Table IV, it will be noted that freeing the line at the distant end, slightly diminishes the current at

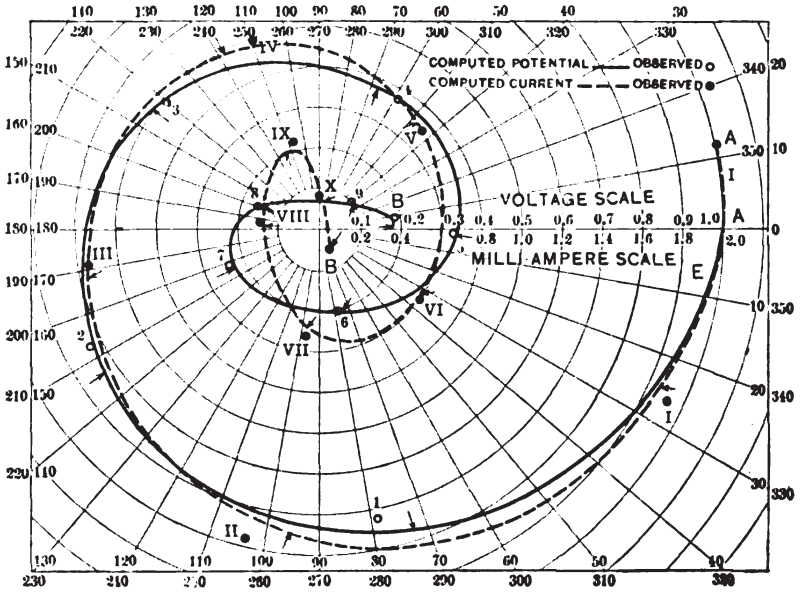

FIG. 5.—LINE GROUNDED THROUGH SUBSCRIBER'S SET. VECTOR DIAGRAM OF VOLTAGE AND CURRENT

the sending end, without materially affecting its phase. Towards the distant end of the line, the magnitude and phase of the current are, of course, increasingly affected.

Figs. 6, 7 and 8 show the computed magnitudes of voltage and current along the imitated line without respect to phase. The broken dotted straight lines indicate the correspondingly computed magnitudes of voltage along the artificial line, on the assumption that the resistance and inductance are uniformly distributed in each section coil; also that the capacitance between the coil windings can be ignored. The voltages at

TABLE VI

ELECTRICAL CONSTANTS OF IMITATED LINE CORRESPONDING TO THE TEN-SECTION ARTIFICIAL LINE AT 760 ~.

	Resistance at 20° cent. ohms	Inductance henrys	Capacitance microfarads	Leakance micromhos	Hyperbolic Angle θ hyps. $\angle$	Surge Impedance z_0 ohms $\angle$	Remarks
Line (800 km.)...	2205	1.173	5.823	0	$12.94\ \underline{/79°.5}$ $= 2.35 + j12.72$ $= 2.35 + j\ \underline{8.10}$	$465\ \angle 11°$	The imitated line corresponds approximately to one of a pair of No. 9 A. W. G. copper wires, diameter 2.9 mm. 0.114" separated in air to a distance of 1000 diameters or 2.9 meters.
Per wire-km.....	2.756	1.463×10^{-3}	7.28×10^{-3}	0	$0.01618\ \underline{/79°.5}$ $= 0.00294 + j0.0159$ $= 0.00294 + j\underline{0.0101}$	$465\ \angle 11°$	
Per loop-km.....	5.512	2.926×10^{-3}	3.64×10^{-3}	0	"	$930\ \angle 11°$	Underscored imaginary quantities are in quadrant measure.
Per wire-mile....	4.44	2.35×10^{-3}	11.72×10^{-3}	0	$0.02604\ \underline{/79°.5}$ $= 0.00473 + j0.0256$ $= 0.00473 + j\underline{0.0163}$	$465\ \angle 11°$	
Per loop-mile....	8.88	4.70×10^{-3}	5.86×10^{-3}	0	"	$930\ \angle 11°$	

section-junctions and line-terminals then agree with those of the imitated line at such points. The current strengths in the artificial line would, if shown on the diagram, be constant in each section, and would be represented by horizontal straight lines, broken by vertical descents at leak-junctions. When multiplied by cosh u, these currents would agree with the imitated line currents at mid-section points.

In the case of Fig. 6 and Table VI, the current strength arriving at the subscriber's set is $0.115 \angle 62°$ milliampere. The potential difference at the terminals of the receiving telephone T was $0.040 \angle 230°$ volt, and taking the impedance of this

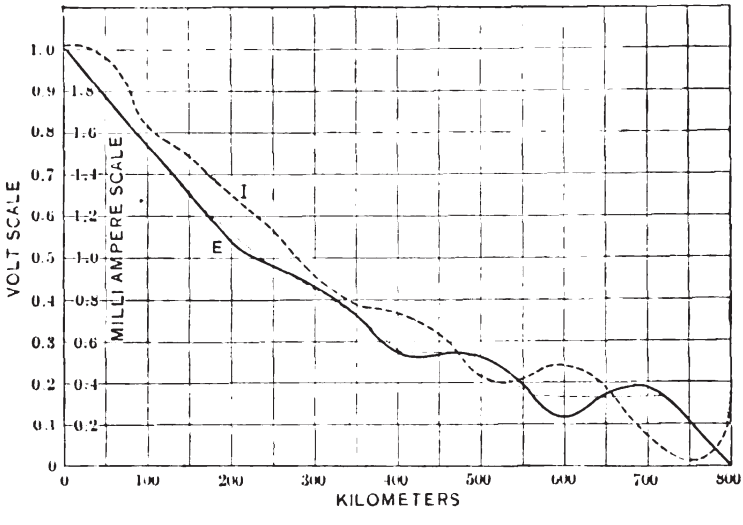

FIG. 6.—AMPLITUDE OF POTENTIAL AND CURRENT ON IMITATED SMOOTH LINE AND AMPLITUDE OF POTENTIAL ON ARTIFICIAL LINE, GROUNDED AT DISTANT END

instrument at $260 \angle 40°$ ohms, as found by subsequent experiments, the current received in the instrument would be $0.040 \angle 230° / 260 \angle 40° = 0.154 \angle 270°$ milliampere. This current gave rise to a loud musical note in the receiver.

EQUIVALENT CIRCUITS OF ARTIFICIAL AND IMITATED LINES

Table VII contains data concerning the artificial line taken as a whole, and also concerning the imitated line, its external counterpart. Figs. 9 and 10 show the nominal and equivalent circuits of these lines. In Fig. 9, the nominal II, $A''B''$, contains an architrave impedance $2202 + j5599$ ohms, and the imitated

line would contain this impedance at the testing frequency. In each of the two terminal leaks there would be 0.01391 /90°.5 mho. corresponding to 2.913 microfarads of total distributed capacity. Similarly, the nominal T, $A'B'$ contains in its arms a total of $2202 + j5599$ ohms, and in its single leak 0.02782 /90°.5 mho.

Fig. 10 shows the equivalent Π, $A''B''$, of three simple impedances which would externally replace either the whole artificial line, or the imitated line. The architrave impedance is $2417\text{-}j\,76$ ohms. Each pillar leak has an admittance of $(1.737 + j\,0.391)\ 10^{-3}$ mho. If we apply one volt to the terminal A'', then by

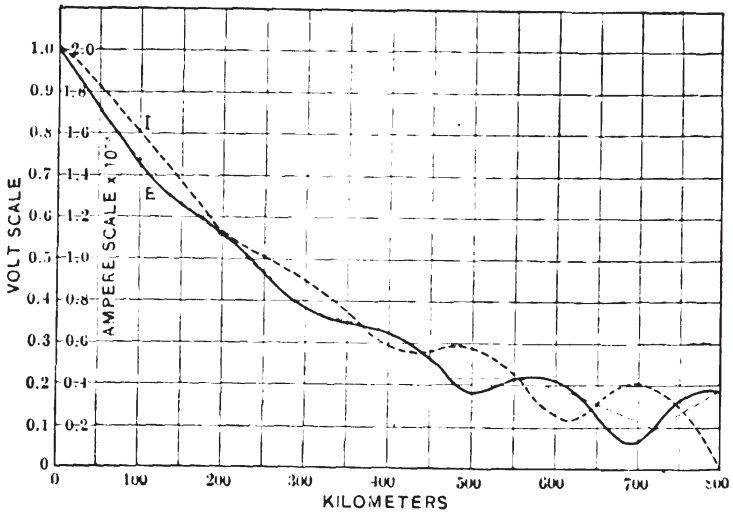

FIG. 7.—AMPLITUDE OF POTENTIAL AND CURRENT ON IMITATED SMOOTH LINE AND AMPLITUDE OF POTENTIAL ON ARTIFICIAL LINE, FREED AT DISTANT END

freeing, grounding, or connecting-to-instrument the end B'', we obtain there the corresponding terminal voltages and currents observed and computed for the artificial line. That is, the current, voltage, and power will be the same at the terminals A'',B'', as at the terminals A and B respectively of the artificial line, or of the imitated smooth line, under like conditions of operation. Similarly, the equivalent T in Fig. 10, has $380 - j62.2$ ohms in each arm, and 11.19×10^{-3} /20°.2 mho. in its leak.

Free and Ground Impedance Tests of Artificial Line. Table VIII collects a set of measurements on the impedance of different lengths; *i.e.*, different numbers of sections in the artificial

line, commencing with one section and ending with 10 sections. Each impedance is the ratio of the terminal impressed voltage to the entering current strength. It will be seen that although the impedances, free and grounded at the far end, vary greatly as the number of sections is changed, yet their product is constant within errors of observation, and its square root, their geometric mean, is the constant surge-impedance of the line. The square root of the ratio of the two impedances also measures the tangent of the hyperbolic angle subtended by the line.

Precision of Measurements. Owing to the great sensitiveness of vibration galvanometers, it is readily possible to measure,

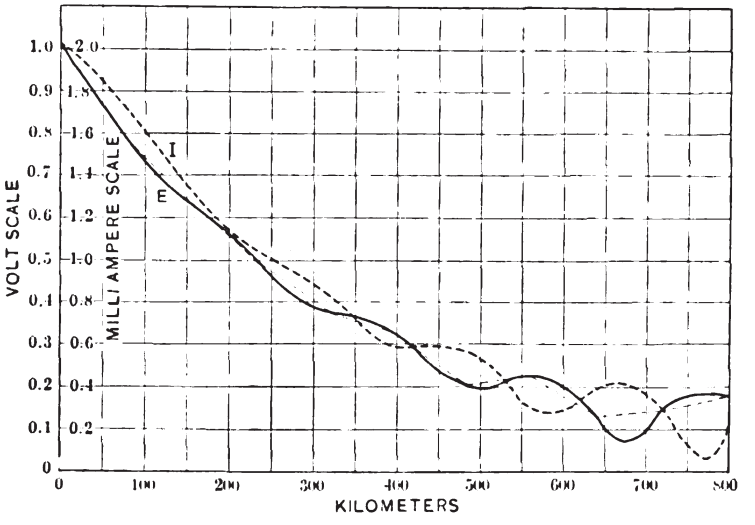

FIG. 8.—AMPLITUDE OF POTENTIAL AND CURRENT ON IMITATED SMOOTH LINE, AND AMPLITUDE OF POTENTIAL ON ARTIFICIAL LINE, GROUNDED THROUGH SUBSCRIBER'S SET AT DISTANT END.

as above described, the potential and current along an artificial line, fairly representing 800 km. (500 miles) of aerial telephone line, without using more than 1.75 volts at the sending end per wire, or 3.5 volts per loop on the metallic circuit basis. The precision of the measurements, however, is not so great at the telephonic frequency of 760 $\sim$, as at the lighting frequency of 60 $\sim$. This relative falling off in precision may be attributed to several causes: namely, first, capacity effects in apparatus and leads not noticeable at low frequencies; second, want of practise with the technique of the method and apparatus at the higher frequency; third, stray magnetic fields in coils and galvan-

TABLE VII

ELECTRICAL CONSTANTS FOR BOTH THE ARTIFICIAL AND IMITATED LINES

Angle θ hyps. $\angle$	Surge impedance z_0 ohms $\angle$	sinh θ	tanh $\theta/2$	Semi-section angle α	Correcting factors		
					cosh α	$\frac{\sinh\theta}{\theta}$	$\frac{\tanh(\theta/2)}{\theta/2}$
$12.94\angle 79°.5$	$465\angle 11°$	$5.2\angle 9°.2$	$0.8277\angle 1°.7$	$0.647\angle 79°.5$	$0.813\angle 5°$	$0.4018\angle 70°.3$	$0.1279\angle 77°.8$
$= 2.35 + j12.72$				$0.1175 + j0.636$			
$= 2.35 + j8.10$				$0.1175 + j0.405$			

Equivalent II

Architrave impedance $z_0 \sinh\theta = 2418\angle 1.8° \text{ ohms.}$

$= 2417 - j76 \text{ ohms.}$

Pillar leak admittance $1/\{z_0 \tanh(\theta/2)\} = 1.78 \times 10^{-3}\angle 12°.7$

Equivalent T

Arm impedance $z_0 \tanh(\theta/2) = 385\angle 9.3° \text{ ohms.}$

$= 380 - j62.2 \text{ ohms.}$

Staff leak admittance $1/(z_0 \sinh\theta) = 11.19 \times 10^{-3}\angle 20°.2 \text{ mho.}$

$= (10.5 + j3.862)\,10^{-3} \text{ mho.}$

ometer leads, a very small flux alternating at the higher frequency, and linked with the galvanometer circuit, being able to set up a very appreciable e.m.f.; fourth, irregularities in the section condensers at the higher frequency (if the condensers along the artificial line are adjusted to equality at a low frequency they may readily show differences as well as diminutions in capacity at the higher frequency); fifth, deviation of the wave form of impressed e.m.f. from the sinusoidal, or the presence of harmonics. This affects the split-phase balance, and phase measurements. The vibration galvanometer may be broadly

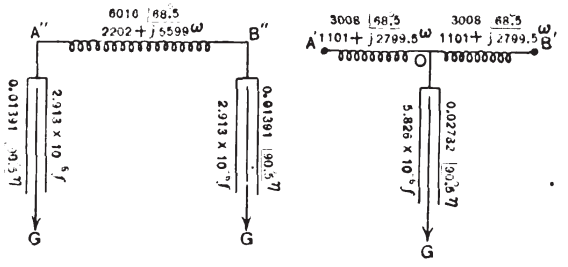

FIG. 9.—NOMINAL Π AND T OF THE IMITATED LINE

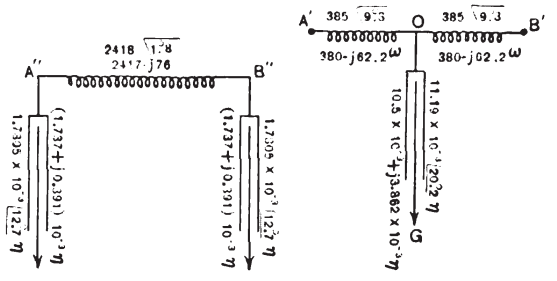

FIG. 10.—EQUIVALENT Π AND T OF ARTIFICIAL AND IMITATED LINES

considered as responding only to the fundamental frequency, the harmonics being independent of it, and unable to affect it. Nevertheless, the presence of the harmonics is able to produce some disturbance of the zero, and thus may have some small influence on the measurements.

By practise, and with further experience, some of the above-mentioned disturbing sources may be reduced, and the precision of measurement increased. Nevertheless, the precision attained in the results here reported will doubtless be considered as satisfactorily sustaining the theory of such telephone-line transmission, at a single telephonic frequency, in the steady

TABLE VIII

IMPEDANCE TESTS OF ARTIFICIAL LINES OF DIFFERENT NUMBERS OF SECTIONS AT 760 ~

No. of sections	Impedance: Grounded Z_g ohms ∠	Impedance: Freed Z_f ohms ∠	$Z_g Z_f$ ohm² ∠	Surge impedance $Z_o = \sqrt{Z_g Z_f}$ ohms ∠	$\tanh n = \sqrt{Z_g / Z_f}$	n	$2u$ mean section angle
1	1244∠38°.4	167\36°	208300\17°.6	456.4\8°.8	2.725 ∠47°.2	0.246 + j1.342	0.246 + j1.342
2	346.4\47°.3	623.8∠28°.2	216100\19°.1	464.9\9°.55	0.7452\37°.75	0.497 + j2.583	0.249 + j1.292
3	441.3∠14°.1	497.8\36°.3	219700\22°.2	468.7\11°.1	0.9415∠25°.2	0.747 + j3.855	0.249 + j1.285
4	588\2°.6	337.5\16°.6	198470\19°.2	445.5\9°.6	1.320 ∠7°.0	0.940 + j4.510	0.235 + j1.128
5	387.7\10°.6	576.1\13°.5	223400\24°.1	472.7\12°.05	0.8203∠1°.45	1.157 + j6.346	0.231 + j1.269
10	449.0\12°.0	476.2\15°	213800\25°	462.4\12°.5	0.9712∠0°.5	2.35 + j12.72	0.235 + j1.272
			Accepted mean values	465.\11°			0.235 + j1.272

state. A great variety of telephonic tests and measurements can be made on such an artificial line under different terminal conditions, and with different receiving instruments. Such an artificial telephone line has great practical value for the telephone engineer, the investigator and the student.

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